Gravitation

Fill in the Blanks

Q.1. The numerical value of the angular velocity of rotation of the earth should be ....rad/s in order to make the effective acceleration due to gravity equal to zero. (1984 - 2 Marks)

Ans. 1.23×10^{-3} rad/s

Solution. We know that $g' = g - R\omega^2 \cos^2 \varphi$

At equator, $\varphi = 0$, Therefore $g' = g - R\omega^2$

Here $g' = 0$ $\therefore \omega = \sqrt{\frac{g}{R}} = 1.23 \times 10^{-3}$ rad/s

Q.2. A geostationary satellite is orbiting the earth at a height of 6 R above the surface of the earth, where R is the radius of the earth. The time period of another satellite at a height of 2.5 R from the surface of the earth is ....hours. (1987 - 2 Marks)

Ans. 8.48 hr

Solution. KEY CONCEPT : According to Kepler's law $T^2 \propto R^3$

$\frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3}$. Here $R_1 = R + 6R = 7R$

and $R_2 = 2.5R + R = 3.5R$

$$\Rightarrow \frac{24 \times 24}{T_2^2} = \frac{7 \times 7 \times 7 \times R^3}{3.5 \times 3.5 \times 3.5 \times R^3} \Rightarrow T_2 = 8.48 \text{ hr}$$

Q.3. The masses and radii of the Earth and the Moon are M_1, R_1 and M_2, R_2 respectively. Their centres are at a distance d apart. The minimum speed with which a particle of mass m should be projected from a point midway between the two centres so as to escape to infinity is .... (1988 - 2 Marks)

Ans. $v = \sqrt{\frac{4G}{d}(M_1 + M_2)}$

Solution. Increase in P.E. of system

$$= \{(P.E.)_i - (P.E.)_f\}$$

$$= -\left\{\left[-\frac{GM_1M_2}{d} - \frac{GM_1m}{d/2} - \frac{GM_2m}{d/2}\right] - \left[-\frac{GM_1M_2}{d}\right]\right\}$$

$$= \frac{Gm}{d/2}(M_1 + M_2)$$

This increase in P.E. is at the expense of K.E. of mass m

$$\therefore \quad \frac{1}{2}mv^2 = \frac{Gm}{d/2}(M_1 + M_2)$$

where v is the velocity with which mass m is projected.

$$\Rightarrow \quad v = \sqrt{\frac{4G}{d}(M_1 + M_2)}$$

Q.4. A particle is projected vertically upwards from the surface of earth (radius R_e) with a kinetic energy equal to half of the minimum value needed for it to escape. The height to which it rises above the surface of earth is.... (1997 - 2 Marks)

Ans. h = R

Solution.

$$\frac{1}{2}mv^2 + \left(-\frac{GMm}{R}\right) = -\frac{GMm}{(R+h)} \qquad \text{...(ii)}$$

From (i) and (ii)

$$\frac{GMm}{2R} - \frac{GMm}{R} = -\frac{GMm}{R+h}$$

or, $-\frac{1}{2R} = -\frac{1}{R+h} \Rightarrow R + h = 2R$

or, $h = R$

True/False

Q.1. It is possible to put an artificial satellite into orbit in such a way that it will always remain directly over New Delhi. (1984 - 2 Marks)

Ans. F

Solution. New Delhi is not on the equatorial plane and geostationary satellite is launched on the equatorial plane.

Subjective Questions

Q.1. Two satellites S_1 and S_2 revolve round a planet in coplanar circular orbits in the same sense. Their periods of revolution are 1 hour and 8 hours respectively. The radius of the orbit of S_1 is 10^4 km. When S2 is closest to S_1, find

(i) the speed of S_2 relative to S_1,

(ii) the angular speed of S_2 as actually observed by an astronaut in S_1.

Ans. (i) $-\pi \times 10^4$ km/hr (ii) 3×10^{-4} rad/s

Soltuion. (i) According to Kepler's third law

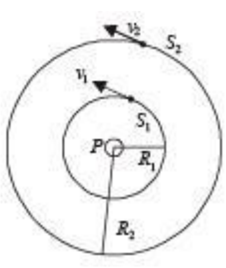
$$\frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3} \Rightarrow R_2^3 = R_1^3 \times \frac{T_2^2}{T_1^2}$$

$$\therefore \quad R_2^3 = \left(10^4\right)^3 \times \frac{8^2}{1^2} = 64 \times 10^{12}$$

$$\Rightarrow \quad R_2 = 4 \times 10^4 \text{km.}$$

Linear speed of satellite S_1

$$v_1 = \frac{2\pi R_1}{T_1} = \frac{2\pi \times 10^4}{1} = 2\pi \times 10^4 \text{ km/hr}$$



Linear speed of satellite S_2,

$$v_2 = \frac{2\pi R_2}{T_2} = \frac{(2\pi)(4 \times 10^4)}{8} = \pi \times 10^4 \text{km/hr}$$

The speed of satellite S_2 w.r.t. S_1

$= v_2 - v_1 = \pi \times 10^4 - 2\pi \times 10^4 = -\pi \times 10^4$ km/hr

(ii) Angular speed of S_2 w.r.t. S_1

$$= \frac{v_r}{R_r} = \frac{v_2 - v_1}{R_2 - R_1} = \frac{3.14 \times 10^4 \times 5/18}{3 \times 10^4 \times 10^3} = 3 \times 10^{-4} \text{ rad/s}$$

Q.2. Three particles, each of mass m, are situated at the vertices of an equilateral triangle of side length a. The only forces acting on the particles are their mutual gravitational forces.

It is desired that each particle moves in a circle while maintaining the original mutual separation a. Find the intial velocity that should be given to each particle and also the time period of the circular motion. (1988 - 5 Marks)

Ans. $\sqrt{\frac{Gm}{a}},\ 2\pi\sqrt{\frac{a^3}{3Gm}}$

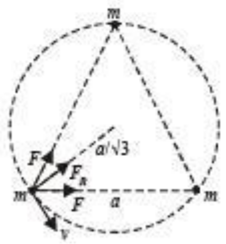
Solution. The radius of the circle

$$r = \frac{2}{3}\sqrt{a^2 - \frac{a^2}{4}} = \frac{a}{\sqrt{3}}$$

Let v be the velocity given. The centripetal force is provided by the resultant gravitational attraction of the two masses.

$$F_R = \sqrt{F^2 + F^2 + 2F^2 \cos 60^\circ}$$
$$= \sqrt{3}\,F = \sqrt{3}\,G\frac{m \times m}{a^2}$$

$$\therefore \quad \sqrt{3}\,G\frac{m^2}{a^2} = \frac{mv^2}{r}$$



$$\left(\frac{mv^2}{r} = \text{centripetal force}\right)$$

$$v^2 = \frac{\sqrt{3}Gmr}{a^2} = \frac{\sqrt{3}Gma}{a^2 \times \sqrt{3}} \Rightarrow v = \sqrt{\frac{Gm}{a}}$$

Time period of circular motion

$$T = \frac{2\pi r}{v} = \frac{2\pi a/\sqrt{3}}{\sqrt{\frac{Gm}{a}}} = 2\pi\sqrt{\frac{a^3}{3Gm}}$$

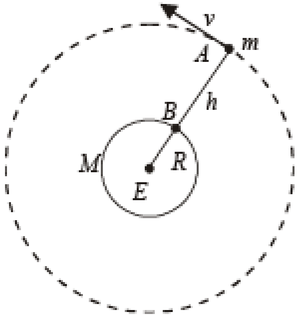
Q.3. An artificial satellite is moving in a circular orbit around the earth with a speed equal to half the magnitude of escape velocity from the earth. (1990 - 8 Mark)

(i) Determine the height of the satellite above the earth's surface.

(ii) If the satellite is stopped suddenly in its orbit and allowed to fall freely onto the earth, find the speed with which it hits the surface of the earth.

Ans. (i) 6400 km (ii) 7.92 km/s

Solution. (i) KEY CONCEPT : Since the satellite is revolving in a circular orbit, the centripetal force is provided by the gravitational pull.



$$\frac{mv^2}{(R+h)} = \frac{GMm}{(R+h)^2}$$

$$\therefore \quad v^2 = \frac{GM}{R+h}$$

$$\text{But } v = \frac{1}{2}v_e = \frac{1}{2}\sqrt{\frac{2GM}{R}}$$

$\therefore \quad \frac{1}{4}\left(\frac{2GM}{R}\right) = \frac{GM}{R+h}$

$\Rightarrow$ 2R + 2h = 4R $\Rightarrow$ h = R = 6400 km.

(ii) KEY CONCEPT : When the satellite is stopped, its kinetic energy is zero. When it falls freely on the Earth, its potential energy decreases and converts into kinetic energy.

$\therefore (P.E.)_A - (P.E.)_B = K.E.$

$$\Rightarrow \quad \frac{-GMm}{2R} - \left(\frac{-GMm}{R}\right) = \frac{1}{2}mv^2$$

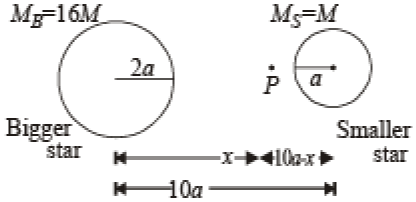
$$\Rightarrow \quad v = \sqrt{\frac{GM}{R}} = \sqrt{gR} = \sqrt{9.8 \times 6.4 \times 10^6}$$

$$= 7920 \text{ m/s} = 7.92 \text{ km/s}$$

Q.4. Distance between the centres of two stars is 10a. The masses of these stars are M and 16M and their radii a and 2a, respectively. A body of mass m is fired straight from the surface of the larger star towards the smaller star. What should be its minimum initial speed to reach the surface of the smaller star? Obtain the expression in terms of G, M and a. (1996 - 5 Marks)

Ans. $\frac{3}{2}\sqrt{\frac{5GM}{a}}$

Solution.



The force of attraction is zero at say x from the bigger star.

Then force on mass m due to bigger star = Force on mass m due to small star

$$\frac{GM_Bm}{x^2} = \frac{GM_Sm}{(10a-x)^2} \quad \Rightarrow \quad \frac{16M}{x^2} = \frac{M}{(10a-x)^2} \Rightarrow x = 8a$$

If we throw a mass m from bigger star giving it such a velocity that is sufficient to bring it to P, then later on due to greater force by the star M_S it will pull it towards itself [without any external energy thereafter].

The energy of the system (of these masses) initially = Final energy when m is at P

$$-\frac{GM_B M_S}{10a}-\frac{GM_B m}{2a}-\frac{GM_S m}{8a}+\frac{1}{2}mv^2$$
$$=-\frac{GM_B M_S}{10a}-\frac{GM_B m}{8a}-\frac{GM_S m}{2a}$$

[$\therefore M_B = 16M$; $M_S = M$]

$$\therefore\ v=\frac{3}{2}\sqrt{\frac{5GM}{a}}$$

Q.5. A body is projected vertically upwards from the bottom of a crater of moon of depth R/100 where R is the radius of moonwith a velocity equal to the escape velocity on the surface of moon. Calculate maximum height attained by the body from the surface of the moon. (2003 - 4 Marks)

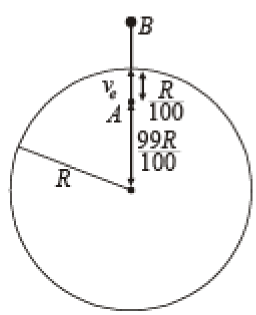
Ans. 99.5 R

Solution. Total energy at A = Total energy at B

$(K.E.)_A + (P.E.)_A = (P.E.)_B$

$$\Rightarrow\ \frac{1}{2}m\times\frac{2GM}{R}+\left[\frac{-GMm}{2R^3}\left\{3R^2-\left(\frac{99R}{100}\right)^2\right\}\right]=-\frac{GMm}{R+h}$$

On solving we get h = 99.5 R.



Integer Value

Q.1. Gravitational acceleration on the surface of a planet is √6/11 g. where g is the gravitational acceleration on the surface of the earth. The average mass density of the planet is 2/3 times that of the earth. If the escape speed on the surface of the earth is taken to be 11 kms^{-1}, the escape speed on the surface of the planet in kms^{-1} will be (2010)

Ans. 3

Solution.

We know that $v = \sqrt{2gR}$

$$\therefore \frac{v_p}{v} = \sqrt{\frac{g_p}{g} \times \frac{R_p}{R}} \quad ...(i)$$

$$\text{Given } \frac{g_p}{g_e} = \frac{\sqrt{6}}{11} \quad ...(ii)$$

$$\text{Also} \quad g = \frac{4}{3}\pi G\rho R \quad \therefore \frac{g_p}{g} = \frac{\rho_p}{\rho} \times \frac{R_p}{R}$$

$$\therefore \frac{\sqrt{6}}{11} = \frac{2}{3} \times \frac{R_p}{R} \qquad \left[\because \frac{\rho_p}{\rho} = \frac{2}{3} \text{(given)}\right]$$

$$\therefore \frac{R_p}{R} = \frac{3\sqrt{6}}{22} \quad ...(iii)$$

$$\text{From (i), (ii) \& (iii)} \quad \frac{v_p}{v} = \sqrt{\frac{\sqrt{6}}{11} \times \frac{3\sqrt{6}}{22}} = \sqrt{\frac{3 \times 6}{11 \times 22}} = \frac{3}{11}$$

$$\therefore v_p = \frac{3}{11} \times v = \frac{3}{11} \times 11 \text{km/s} = 3 \text{km/s}$$

Q.2. A bullet is fired vertically upwards with velocity v from the surface of a spherical planet. When it reaches its maximum height, its acceleration due to the planet's gravity is 1/4 th of its value of the surface of the planet. If the escape velocity from the planet is $v_{esc} = v\sqrt{N}$, then the value of N is (ignore energy loss due to atmosphere) (JEE Adv. 2015)

Ans. 2

Solution. Let h be the height to which the bullet rises then, $g^1 = g\left(1+\frac{h}{R}\right)^{-2}$

$\Rightarrow \quad \frac{g}{4} = g\left(1+\frac{h}{R}\right)^{-2}$

$\Rightarrow \quad h = R$

We know that $v_e = \sqrt{\frac{2GM}{R}} = v\sqrt{N}$ (given) ...(i)

Now applying conservation of energy for the throw

Loss of kinetic energy = Gain in gravitaional potential energy

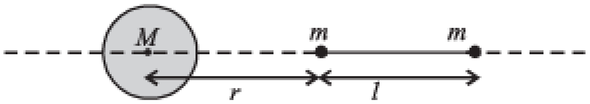
$\therefore \quad \frac{1}{2}mv^2 = -\frac{GMm}{2R} - \left(-\frac{GMm}{R}\right)$

$\therefore \quad v = \sqrt{\frac{GM}{R}}$ ...(ii)

Comparing (i) & (ii) N = 2

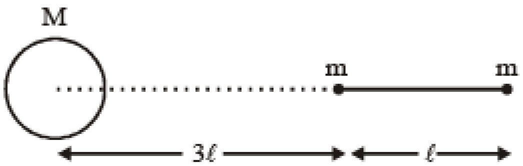
Q.3. A large spherical mass M is fixed at one position and two identical point masses m are kept on a line passing through the centre of M (see figure). The point masses are connected by a rigid massless rod of length ℓ and this assembly is free to move along the line connecting them. All three masses interact only

through their mutual gravitational interaction.
When the point mass nearer to M is at a distance r = 3ℓ from M, the tension in the rod is zero for $m = k\left(\frac{M}{288}\right)$**. The valueof k is** **(JEE Adv. 2015)**



Ans. 7

Solution. For the tension in the rod to be zero, the force on both the masses m and m should be equal in magnitude and direction. Therefore



$$\frac{GMm}{(4\ell)^2}+\frac{Gmm}{\ell^2}=\frac{GMm}{(3\ell)^2}-\frac{Gmm}{\ell^2}$$

$\therefore \quad 2m = M\left[\frac{1}{9}-\frac{1}{16}\right]$

$\therefore \quad m = \frac{7M}{288}$

$K = 7$